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# Summary of dynamics of the regular heptadecagon: $N = 17$

This is the third super-symmetric  $4k+1$  prime polygon, so there are sequences of Dads and Moms which converge to GenStar and the ratio of the periods approaches 18.

First Family

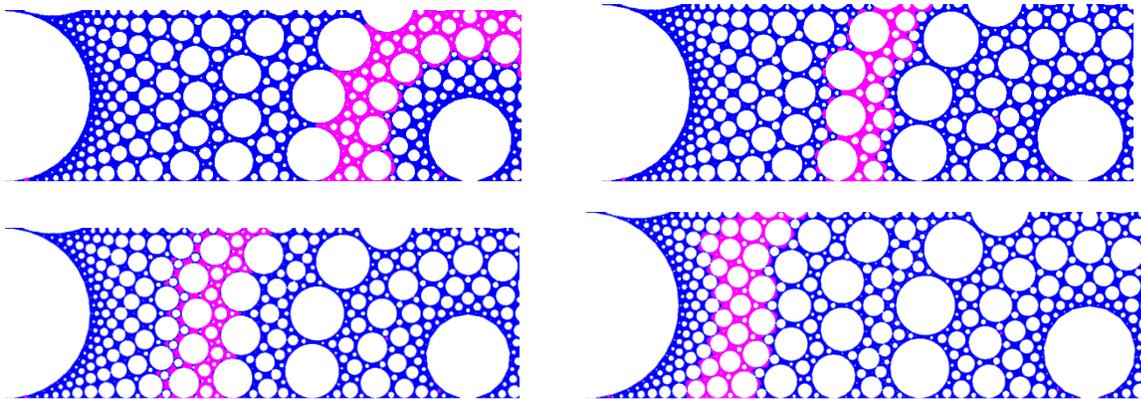


As  $N$  increases the First Families are torn apart to go with either Mom or Dad.

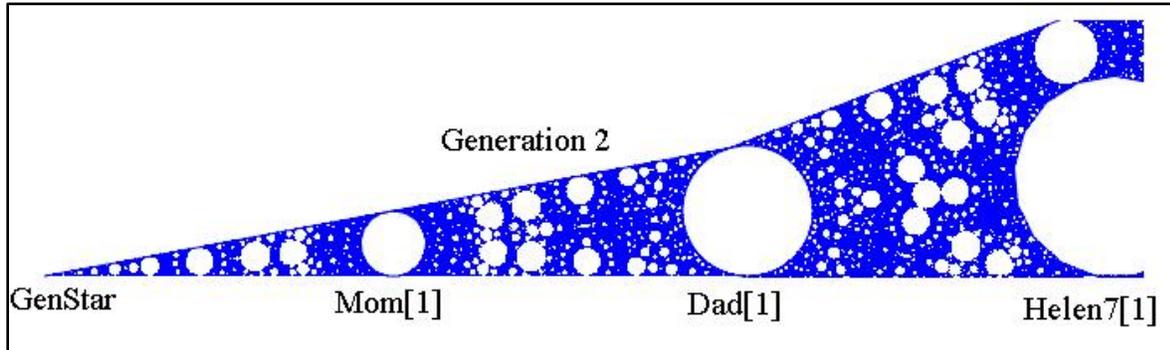
scale[1]	1.0
scale[2]	0.4825281394557327687808777306
scale[3]	0.30190618744782540928550769589
scale[4]	0.20505507150452006840695842796
scale[5]	0.14116479357545073942713885861
scale[6]	0.093081273699400389065240354948
scale[7]	0.053186852624513878400563550418
scale[8] (GenScale)	0.0173218375167882251854063312595

GenStar:  $\{-10.6079691394450130883380556195, -0.982973099683901778281948844855\}$

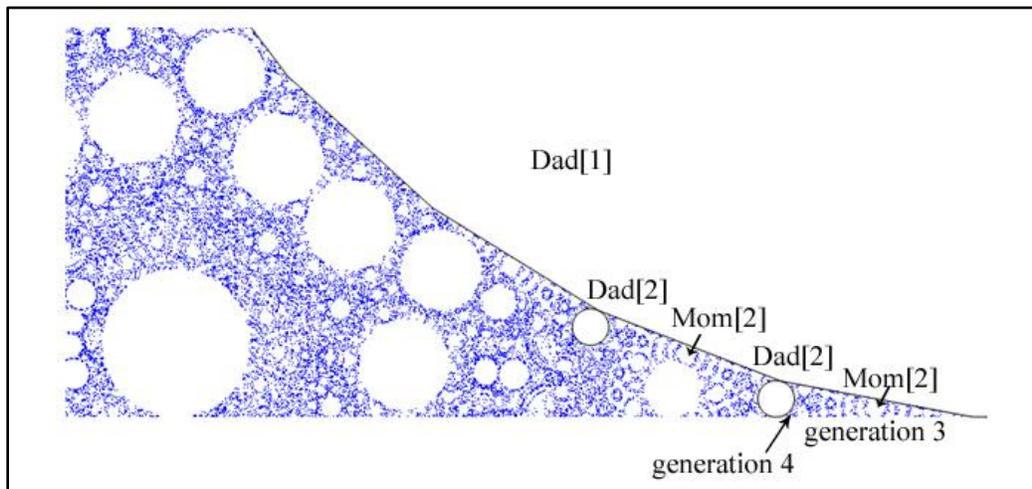
Below are web plots showing the 6 invariant regions.



Below is generation 2. At this scale it is hard to see the tiny Dad[2] buds that surround Dad[1]. Based on the results from  $N = 13$ , we can expect to find odd-even dichotomy starting with generation 2.



The plot below shows generation 3 under Dad[1] - which is identical to the 'canonical' generation 3 at GenStar. The generations will alternate sides, so generation 2 above is on the right side under Dad and generation 3 below is on the left side under Dad[1]. Generation 4 will be on the right of Dad[2] as shown below.



We would expect to find the same dichotomy of even and odd generations that occurred with  $N = 13$ , so the new generation 4 should be self-similar to generation 2 - although there is not yet enough detail to support this hypothesis. There is enough detail above to see that Generation 3 is not a match for generation 2 but it may be self-similar to generation 5.

Periods of Moms:  $\text{CFR}[r_] := (1-r) * \text{GenStar}$ ;  $\text{cMom}[k] := \text{CFR}[\text{GenScale}^k]$ ;

Mom[k]	Period
Mom[1]	238
Mom[2]	5780
Mom[3]	102544
Mom[4]	1847288
Mom[5]	55249688
Mom[6]	598495880

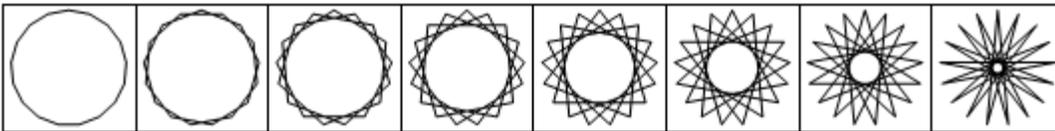
Ratios:  $25 \rightarrow 17.7 \rightarrow 18.0145 \rightarrow 17.9992 \rightarrow 18.00004499$

Just like  $N = 13$ , the orbit of these Mom (or Dads) are highly localized and they tell us nothing about the fractal dimension - which is probably multi-fractal given that there are 7 non-trivial scales.

## Projections

$N = 17$  has 8 projections as shown below

`GraphicsGrid[{Table[Graphics[poly[Wc[[k]]]],{k,1,8}],Frame->All]`



As with all regular  $N$ -gons, there is a contrast between inner star projections and outer star projections, because the outer star projections show the effects of extra rotations in their orbits. We have methods of 'filtering' projections, but we will not do this here. The inner star projections tend to be light and delicate and many of the outer star projections are heavy with 'embellishments'. We will illustrate this with two points, both of which have orbits which may be non-periodic.

**Example 1:** An inner star orbit:  $q_1 = \{-0.67162759883690885347478, -0.79976832146607331\}$ ; Below are samples of all 8 projections - starting with the 'null' projection P1 which is the  $\tau^2$  orbit of  $q_1$ . We will plot the P1 projection as points and the remaining projections as lines. Mom is shown in each one for scale, but sometimes she is obscured by the plot.

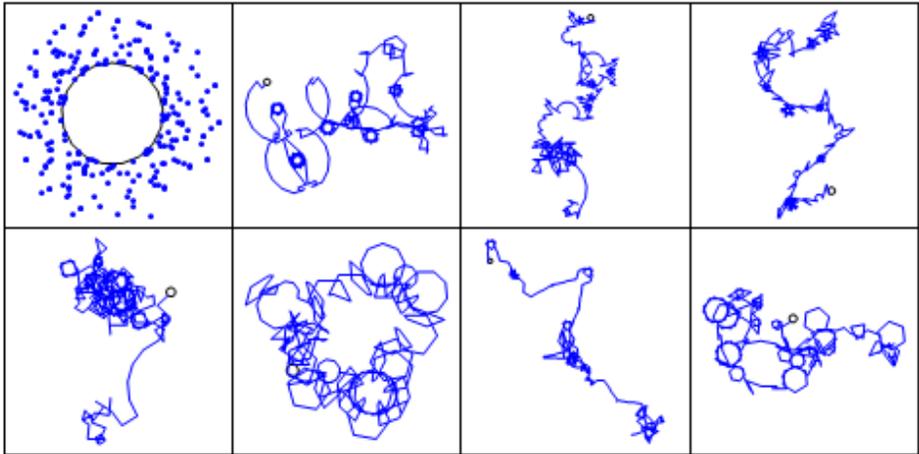
**Ind = IND[q1, 20000]; k = 250;** (\*to plot the first 250 points in the projections\*)

**Px=Table[Graphics[{poly[Mom],Blue,Line[PIM[q1,k,j]]}],{j,1,HalfN}];**

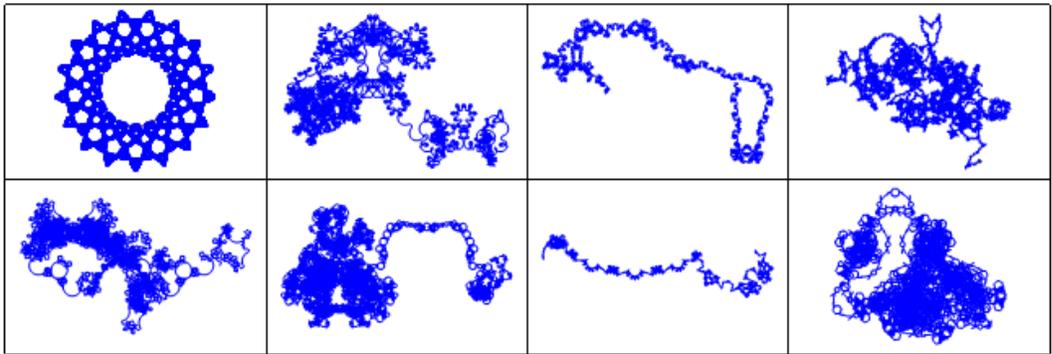
**Px[[1]]=Graphics[{poly[Mom],AbsolutePointSize[3.0],Blue,Point[PIM[q1,k,1]]};**

(\*plot the P1 projection with points instead of lines\*)

**GraphicsGrid[{{Px[[1]],Px[[2]],Px[[3]],Px[[4]]}, {Px[[5]],{Px[[6]],Px[[7]], Px[[8]]}},Frame->All] (\*two groups of four each\*)**

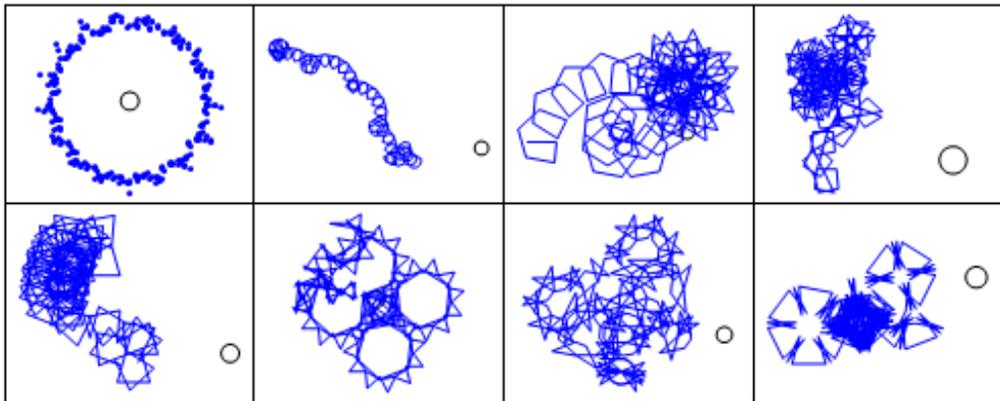


Below is **k = 10,000**

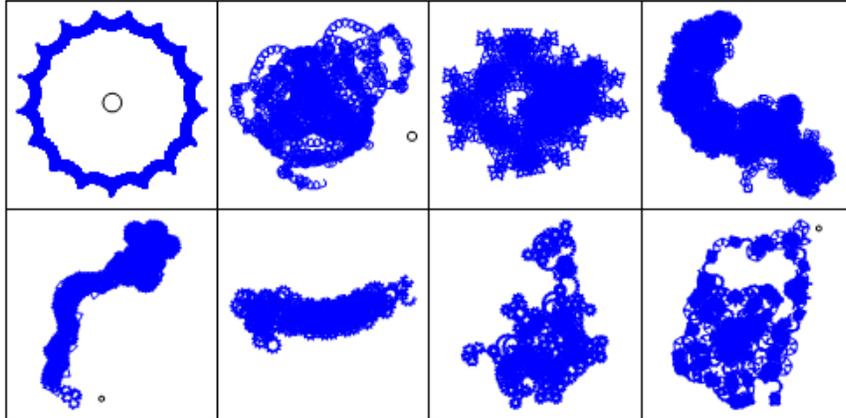


**Example 2:** All 8 projections for an an outer star orbit from generation 2:  
 $q1 = \{-10.16261085024686410852673, -0.9492630849924690705479934\};$

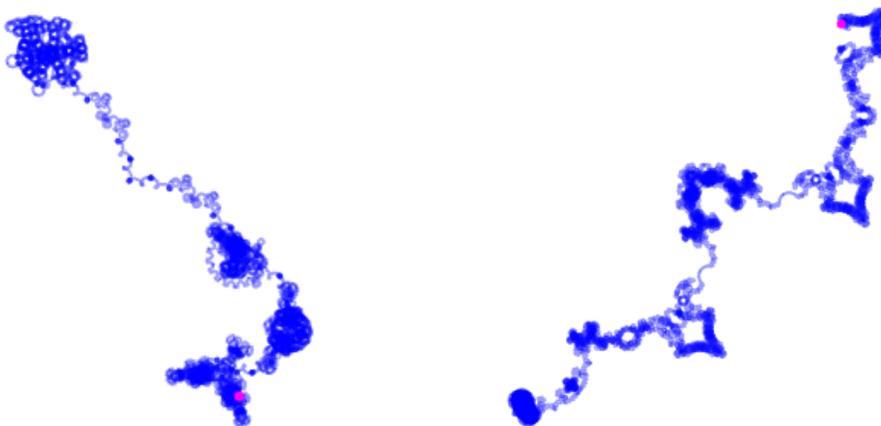
**k = 250**



**k = 10,000** below

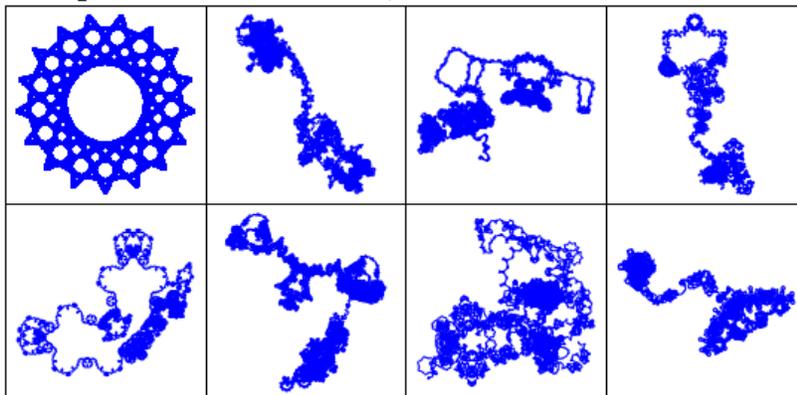


Even though these outer star projections seem very 'thick', their large-scale structure is beautiful and the plots show clearly the large-scale symmetry of the orbit. Below are the first 400,000 points for P2 and P6. The magenta dot is the initial point  $q_1$ . P2 is only 300 units high but P6 is 900 units.

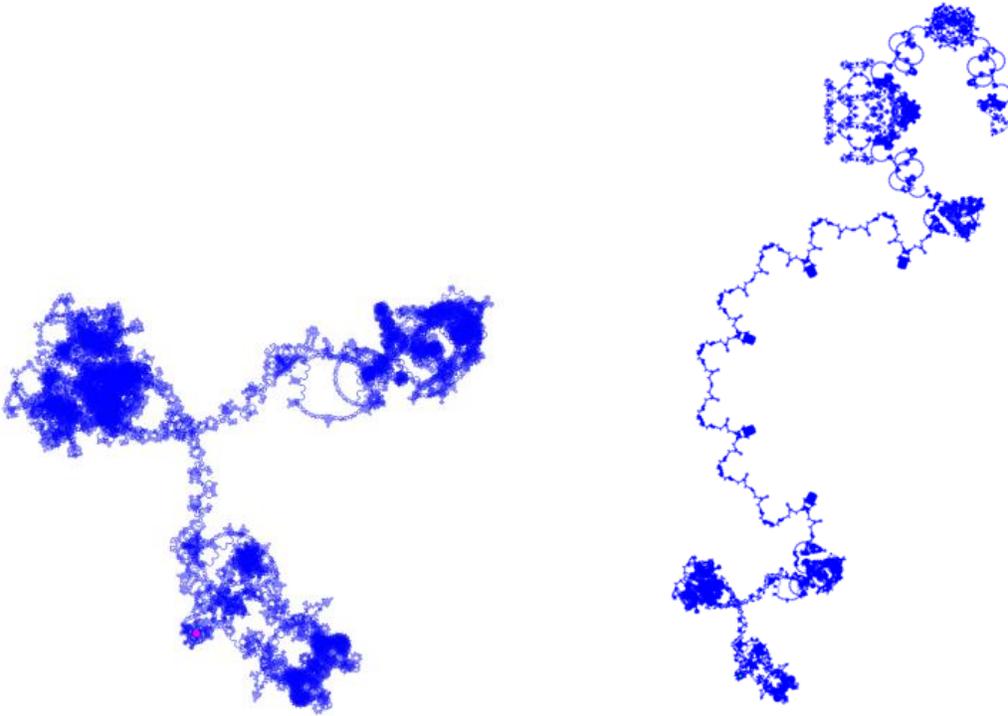


The dynamics of the inner star region are very complex and the projections have a amazing diversity. Below are longer orbital projections for  $q_1 = \{-0.67162759883690885347478, -0.79976832146607331\}$  which is the same point used above.

**Example 3: Inner star k = 200,000**



Below is P2 for  $k = 400,000$  and on the right is its Pinwheel continuation (see next section).



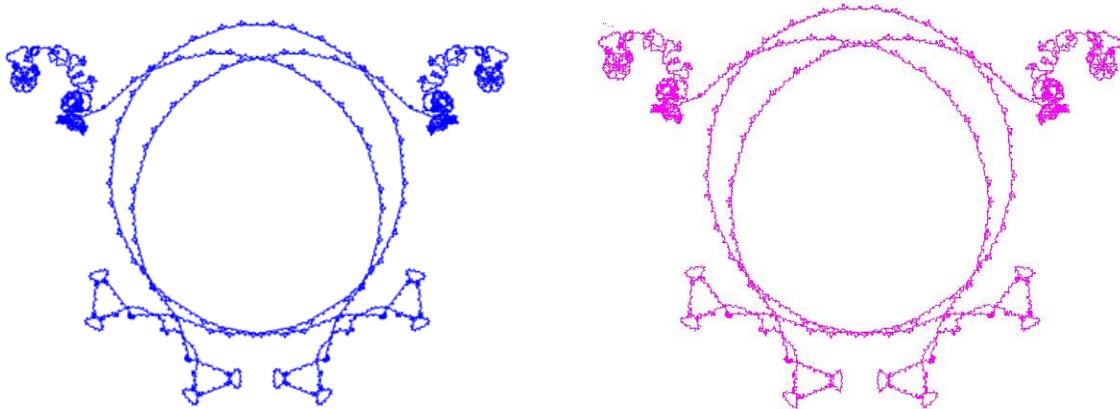
### Interior Projections vs. Pinwheel Projections

For Pinwheel projections we will move out to the third ring of Dads to make sure that every orbit visits all 34 of the 'strips' surrounding Mom. The local MomRing3 origin is  $cMomRing3 = \{-6 * cS[8][[1]], 0\}$  where  $cS[8][[1]]$  is the x coordinate of Dad's center, so  $cMomRing3 = \{64.750311943569500519474786755, 0\}$ .

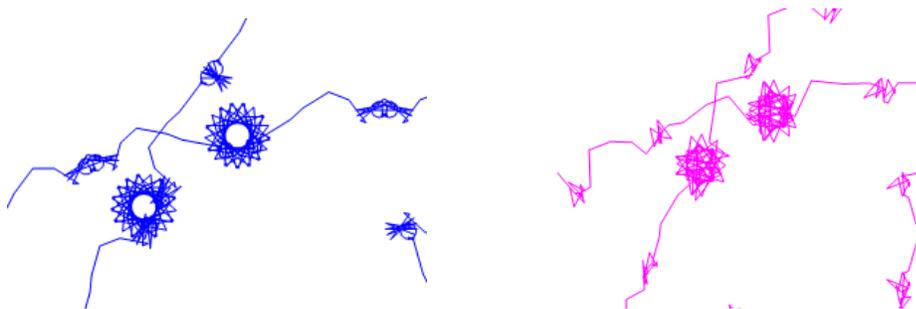
On a large scale the Pinwheel projections appear to be identical to the interior projections obtained above, but on a small scale, the interior projections have extra 'embellishments' because they do not filter out rotations around Mom. The Pinwheel projections are based on points which return to a single 'strip' so they ignore the remaining points in the orbit. Since these remaining points are essentially rotations, there is little information lost.

#### Example 4: Interior vs. Pinwheel

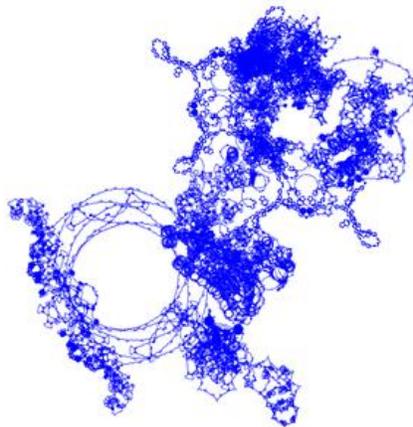
To generate the Interior projection:  $q1 = \{-1.890275620948204, 0.4155330000\}$ ;  
**Ind = IND[q1, 1000000]; P7 = PIM[q1, 500000, 7]; D3 = Drop[P7, -217000];**  
**D2 = Drop[D3, 247000];** (\*Resulting D2 contains just 36,000 points shown in blue below\*)  
**Graphics[{Blue, Line[D2]}]**



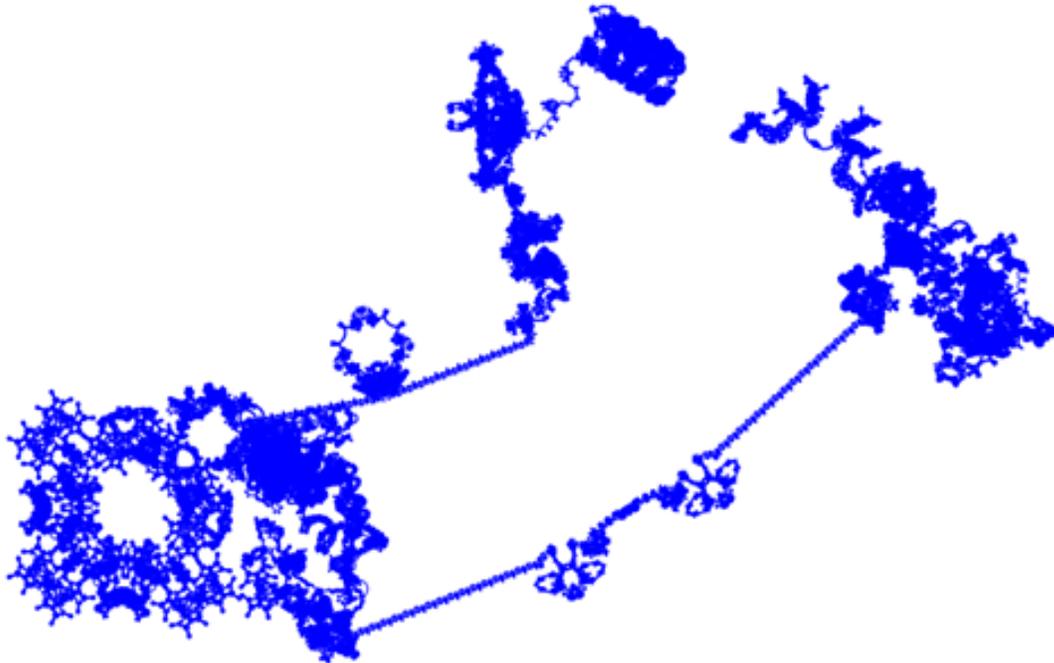
The corresponding magenta Pinwheel projection appears to be identical but the detail is quite different as we can see below:



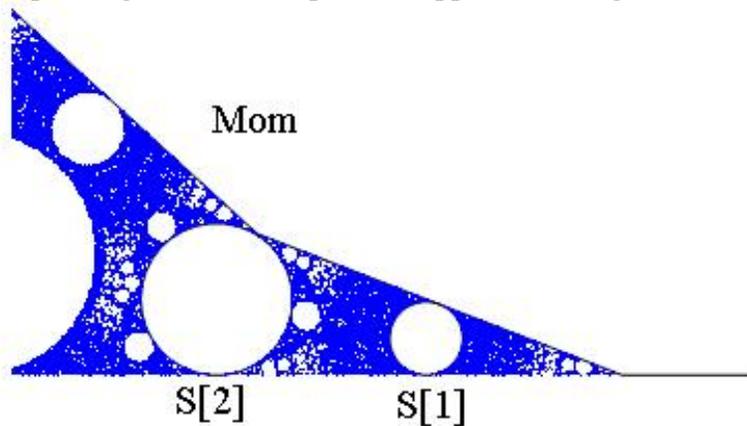
This plot was extracted from the original P7 shown below



The Pinwheel P2 projection below covers about 2 million points so it took several days to generate (about 400 million iterations using  $p1 = \{63.481506479270908, -0.2008962228195\}$ ). This is known as the 'fruit basket'. The scale is 10,000 wide by 8,000 high. It runs from top center to far right. The symmetric episodes are followed by very irregular behavior.

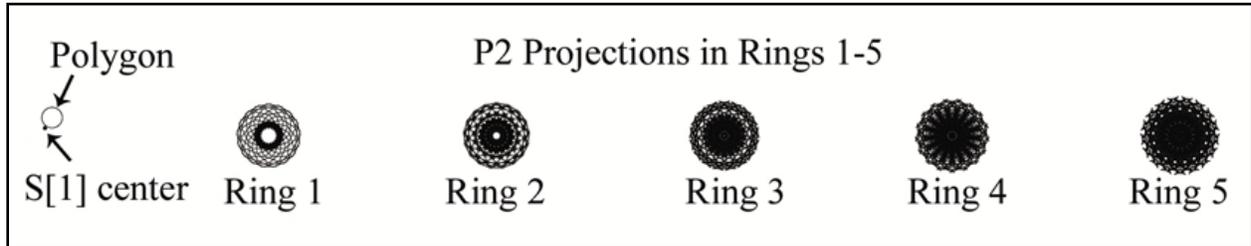


Below are the corresponding 2 million P1 points cropped to the region around Mom.



## Exploring the Outer Rings

The interior projections will function anywhere, even though we usually apply them to points in the star region. Every point inside Ring 1 has a 'clone' in Ring[k] and we will compare the P2 projections for the first 5 rings using  $cS[1]$  - the center of the step-1 bud as shown by the arrow below. The Pinwheel projections do not change as we pass from the inner rings to the outer rings, but the interior projections become much more dense because of the extra rotations around Mom. We can see this clearly in the plot below.



Here are the detailed plots - starting with Ring 0 and Ring 1. The magenta dot is the initial point.

