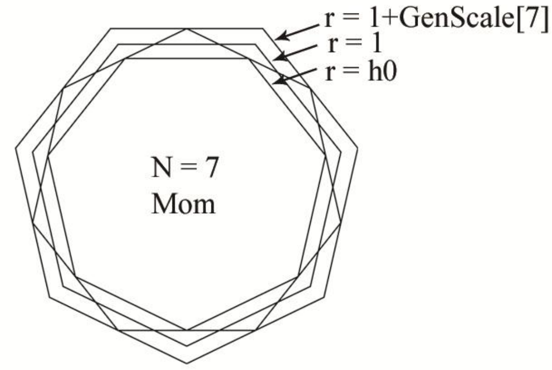
DynamicsOfPolygons.org

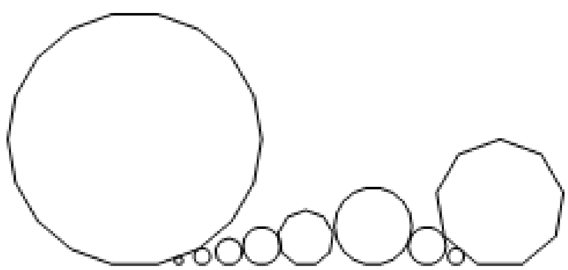
Woven polygons

Woven Polygons

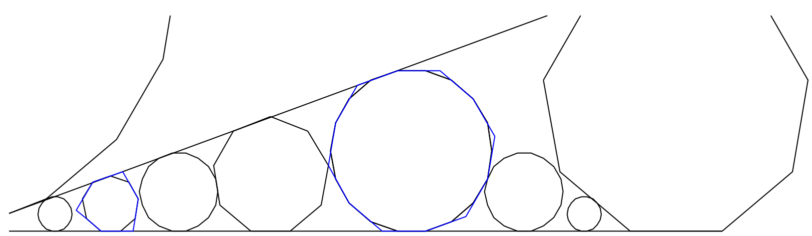
We define a woven polygon to be the 2N-gon formed by alternating vertices from a regular N-gon at radius 1 (Mom) and a secondary regular N-gon at radius between h0 (height of Mom) and 1 + GenScale[N]. This covers the full range of convex proportions.



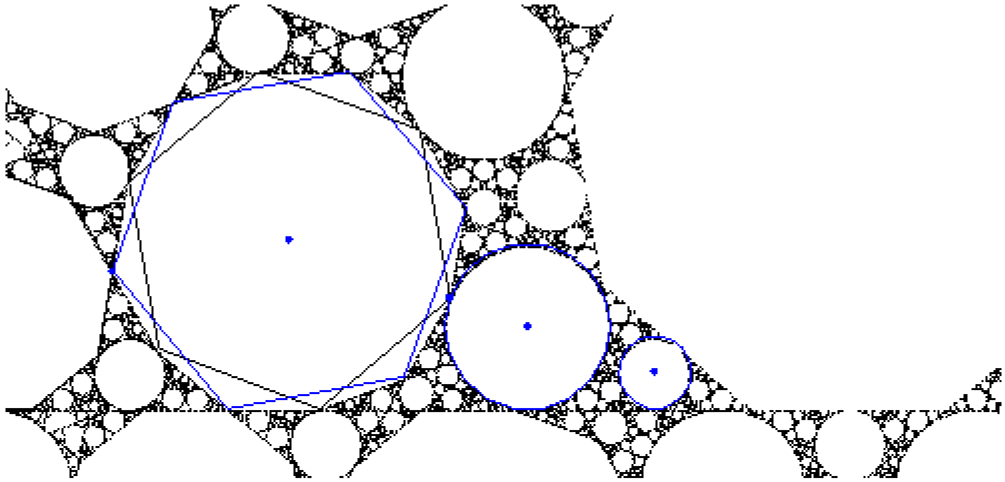
Polygons such as these arise naturally under the Tangent map τ (also known as the outer billards map). For example the regular ennagon (N = 9) is the first odd with a non-trivial divisor and this creates 'mutations' in the First Family.A ‘normal’ N = 9 First Family would like the Mathematica plot below:



But the actual First Family has two mutations - S[3] and DS[3] as shown in blue below

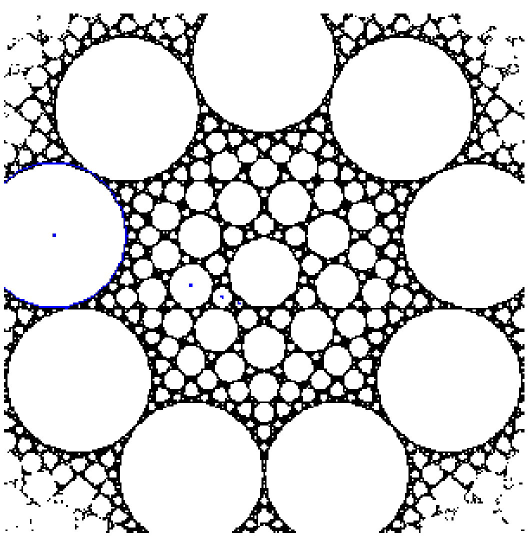


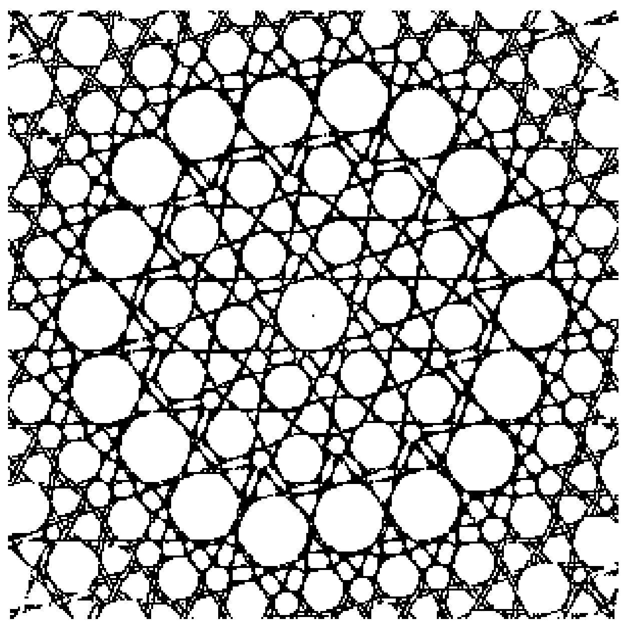
The DS[3] (Dad step-3) bud on the left above, would normally be a regular 9-gon but it now has extended edges to form a non-regular hexagon. On the right, the S[3] (step-3) bud would normally be a regular 18-gon, but the divisor of 3 has partitioned the dynamics into two woven hexagons as shown below.



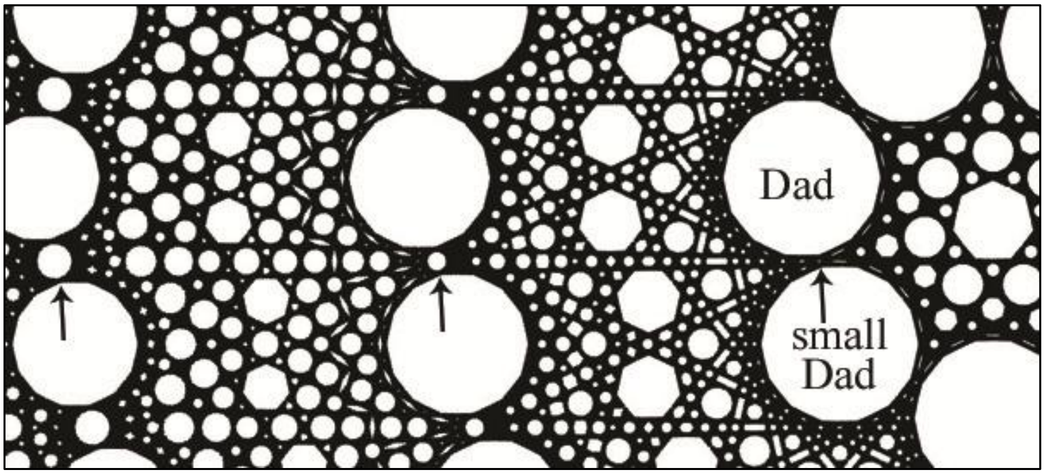
We can find the exact parameters of the two hexagons because they share vertices with S[2] on the right and DS[5] on the left.The blue hexagon above has radius equal to the distance from vertex 3 of DS[5] to the S[3] center and the black hexagon is constructed in the same fashion using vertex 11 of S[2]. The ratio of the radii is 0.954188894138671133499268364

We put this 12-gon at the origin to see what the resulting dynamics would be and found that 'channels' form allowing points from the star region to migrate outwards (and conversely). This led to a study of these 'woven' polygons but it is not clear whether these channels allow for unbounded orbits. Below is the N = 9 web on the left and the S[3] web on the right.



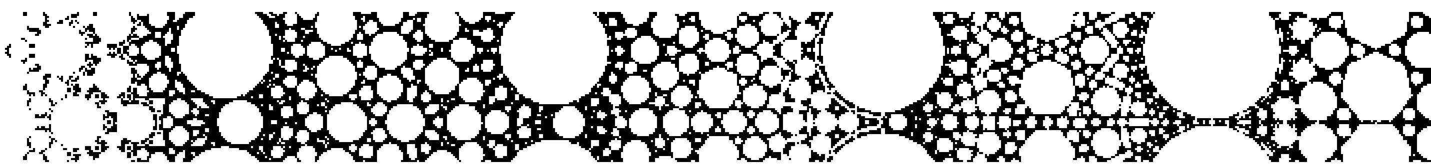


Example 1: For N = 7, the range is 0.900968867 (h0) to 1.109916264174 (1 + GenScale[7]).
Show below is index .91

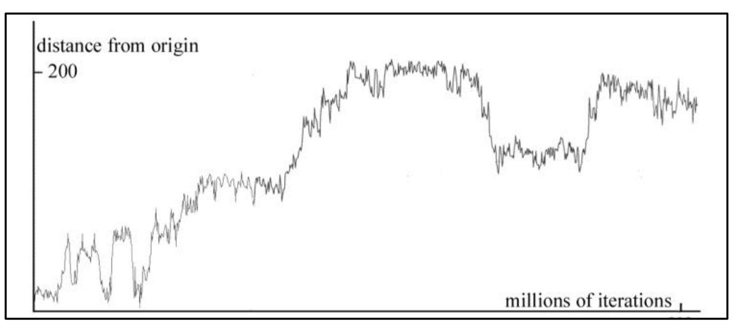


The normal ring of 14 Dads is now subdivided. The orbit of the 7 large Dads 'sees' only the canonical N = 7 Mom and the 7 small Dads (which are almost invisible above) , 'see' only the secondary Mom. (For index 1 this is the canonical N = 14 case.)

The star region is no longer invariant. Rings of Dads undergo periodic oscillations. In the first cycle the secondary Dads (shown at the arrows above and in the 'strip' below) grow and become dominant.



The first Big Crunch at ring 11 sees the canonical Dads almost disappear. Inter-ring spacing (and periods) are same as N = 7, so ring 11 is at radial distance of about 91.6. (The exact parameters of the Crunches are easy to calculate but they do not generally correspond to ring centers.) The second Big Crunch at about ring 22 involves Moms. The third Big Crunch at ring 32 is a close repetition of the first so complete cycle is about 22 rings.
Dynamics are very complex. Many orbits diverge rapidly at first and then fluctuate widely in distance.
The plot show here is the first 900 million points of an 'energetic' point.

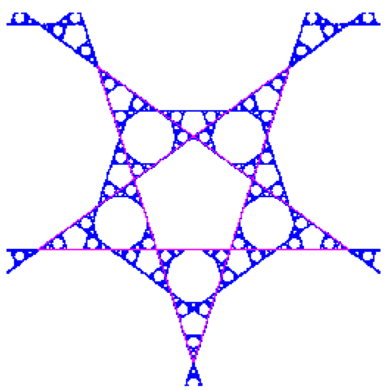


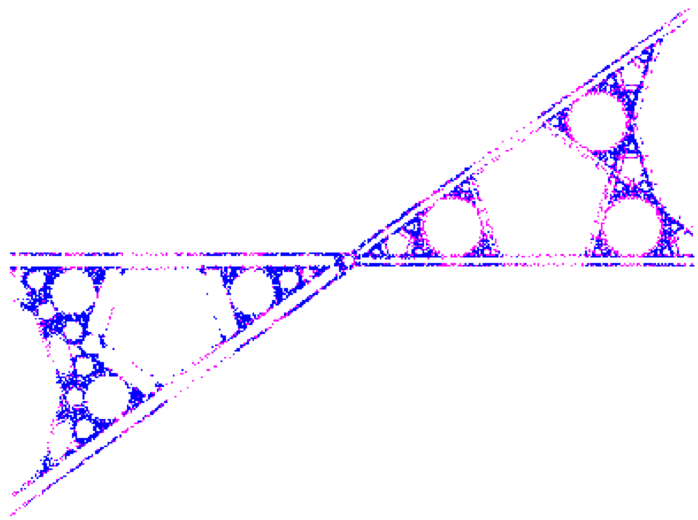
Example 2; The Regular Pentagon

For N = 5, the index range is 0.809016994374 to 1.236067977499. The dynamics are similar to the heptagon, but the pentagon has its own unique dynamics. To get an 'almost regular' polygon, use an index close to h0. Below is index .81

S = Mom; T = RotateCorner[{0, 0} - {0, .810000000000000000000000000000}, 5, {0, 0}];
(*extra zeros ask for extra precision*)
W = {S[[1]], T[[4]], S[[2]], T[[5]], S[[3]], T[[1]], S[[4]], T[[2]], S[[5]], T[[3]]} (*this is the 'weave*)

As expected, the web looks very much like the canonical web, but there are small 'escape' channels which are outlined here in magenta.





Some points do diverge but the rate of divergence is hampered by rotations and the small channels. Below is a typical distance plot for 120 million iterations.