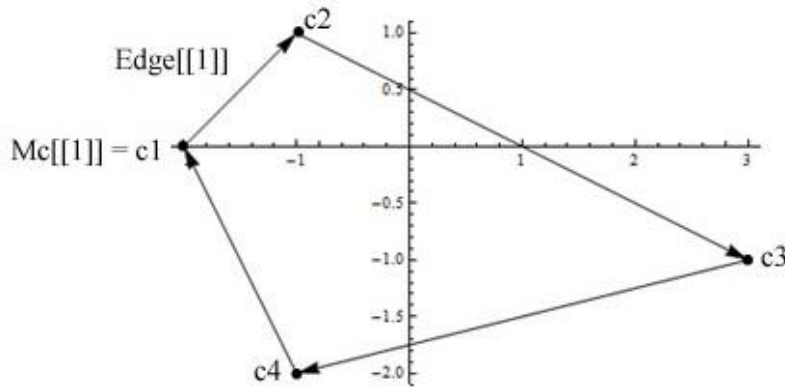


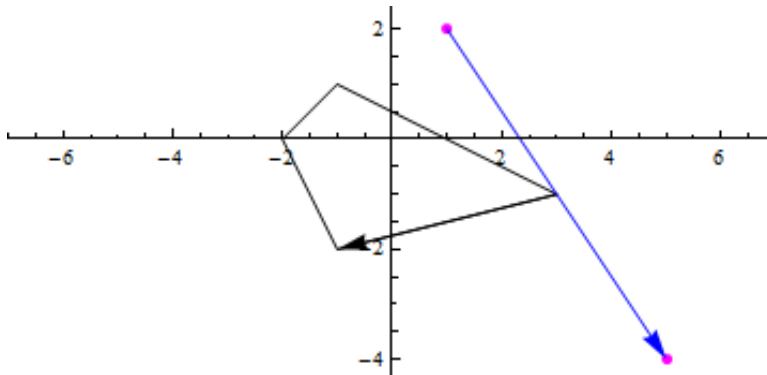
GeneticsOfPolygons.org

Summary of dynamics of a non-regular lattice polygon: $N = 4$

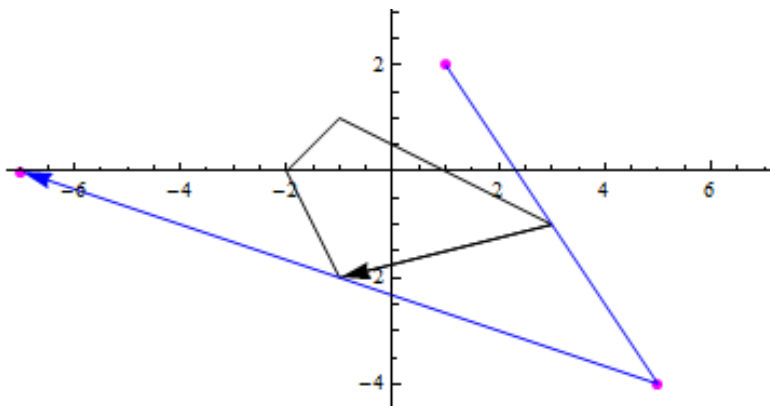
Using the notebook NonRegular.nb: $Mc = \{-2, 0\}, \{-1, 1\}, \{3, -1\}, \{-1, -2\}, \{-2, 0\}$



Because this is a lattice polygon it is easy to compute orbits manually. For example $q = \{1, 2\}$ maps to $\tau(q) = -q + 2c_3 = -\{1, 2\} + 2\{3, -1\} = \{5, -4\}$ as shown below



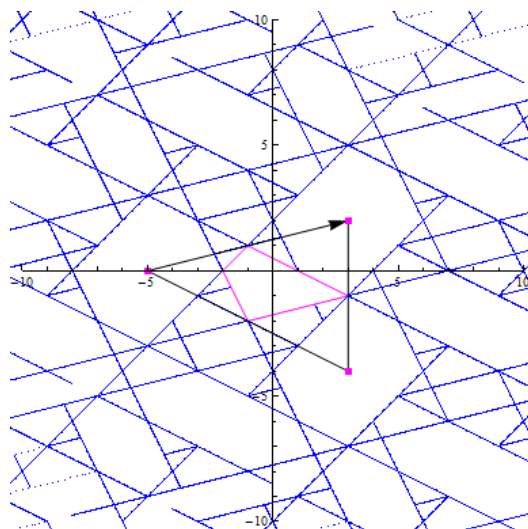
Continuing with this same orbit: $\tau(\{5, -4\}) = -\{5, -4\} + 2c_4 = \{-7, 0\}$



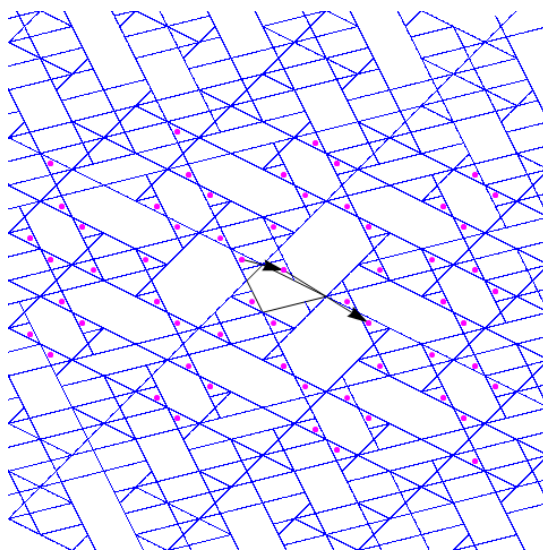
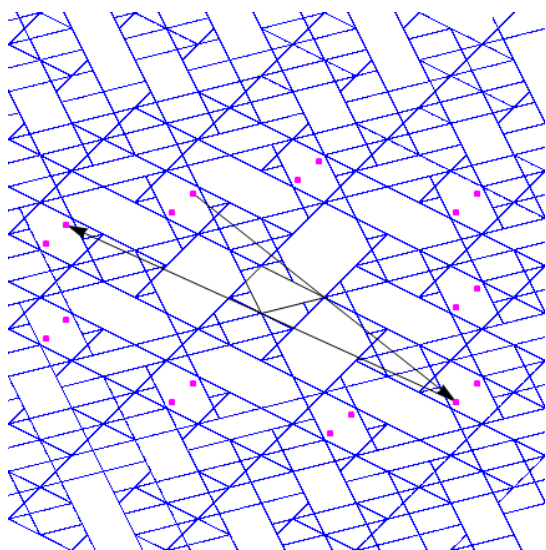
Note that $\tau^2(q) = q + 2(c_4 - c_3)$. This is called the 'return' map and it is always has this form.

Below is the level-300 (forward) web

```
box[{0,0},10]; Show[Graphics[{{AbsolutePointSize[1.0],Blue,Point[WebPoints[.01, 12, 300]],
Magenta,poly[Mom]}},Axes->True,PlotRange->{{left,right},{bottom,top}}]]
```

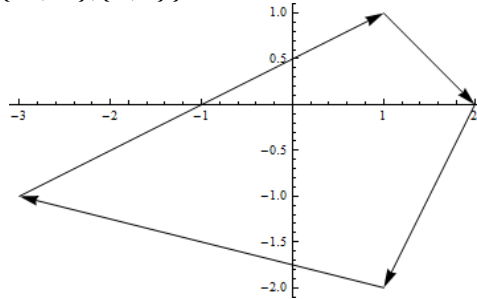


The orbit shown above is $q_1 = \{-5,0\}$ and it has period 3. The orbit is $\{\{-5,0\},\{3,2\},\{3,-4\}\}$. This is the center point of these regions. The remaining points in these regions have period 6. These three regions are similar to a ring of 'Dads'. There is a second ring of 'Dads' as shown on the left below. The centers have period 9 and the remaining points have period 18. All regions with odd period have this type of period doubling. On the right below is a region with period 60. Since this is even, there is no doubling. This means there are 60 congruent regions - each one being an inversion of the previous.



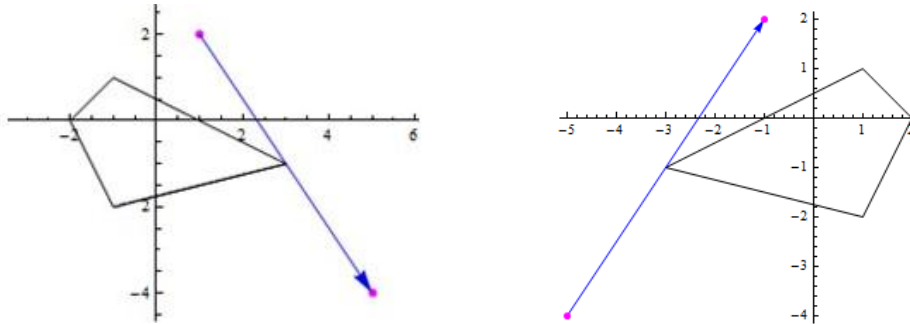
Every convex polygon has an 'inverse' obtained by reflection about the y axis. The inverse has the same dynamics but with reverse orientation. This allows us to generate the inverse web for the original polygon.

The inverse polygon for M above is **Reverse[ReflectionTransform[{1,0}][M]]**
 $=\{2,0\},\{1,-2\},\{-3,-1\},\{1,1\}$ as shown below

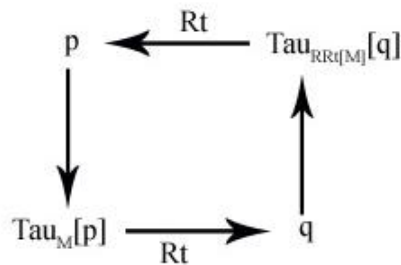


We call this polygon the 'inverse' of M. It has the 'same' dynamics as M but the new τ is τ^{-1} relative to M.

For example, using M, **Tau[{1,2}]** = $-\{1,2\} + 2M[[2]] = \{5,-4\}$ as shown on the left below. On the right is the 'same' map in reverse: **ReflectionTransform[{1,0}][{5,-4}]** = $\{-5,-4\}$ and this maps to **Tau[-5,-4]** = $-\{-5,-4\} + 2RRt[M][[3]] = \{-1,2\}$ which is **ReflectionTransform[{1,0}][{1,2}]**.

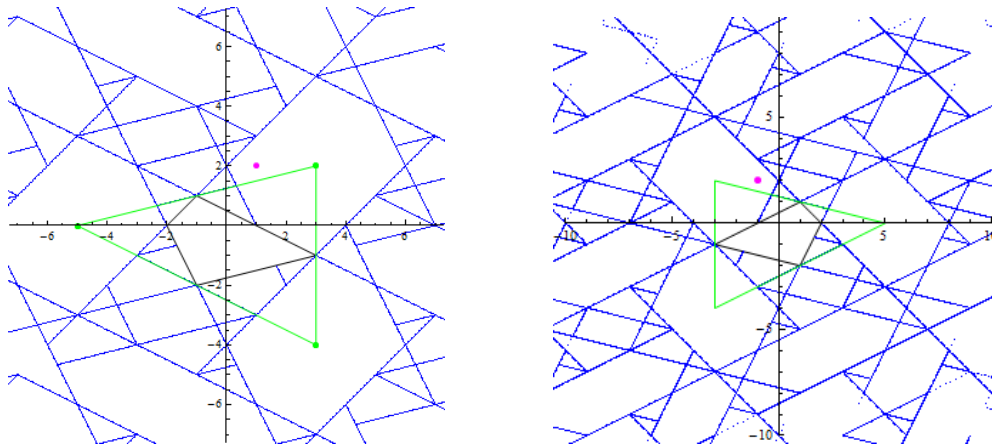


The following diagram commutes:



The two polygons have reflected dynamics and reflected webs.

On the left below is the web for M showing the point $\{1,2\}$ in magenta and the period 3 orbit of the point $\{-5,0\}$ in green. On the right is the web for $RRt[M]$ showing the matching point $\{-1,2\}$ in magenta and the 'same' green orbit but with $\{5,0\}$ as the initial point.



If we invert the web on the right, it will give us the inverse web for M . This web is similar to the forward web, but it gives the correct subdomains at each iteration. When the webs are no longer evolving, the forward and inverse webs are identical, so we will see no difference here.