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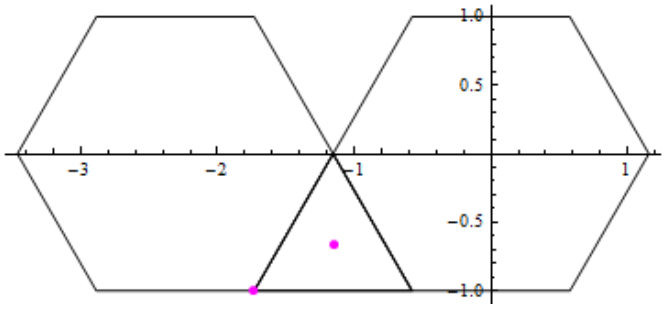
Summary of dynamics of the regular haxagon: N = 6

Since N is 'twice-prime' its dynamics can be derived from N = 3. They share the same web structure but the periods are different because 'Dad' is now the central polygon instead of 'Mom.

The First Family



By convention Dad's height is 1, so rDad ≈ 1.154700538379251529018297561 00
and rMom = 2/3; GenStar ≈ {-1.732050807568877293527446341 51,-1.0}
center of Mom = rMom*GenStar = (2/3)*GenStar ≈ {-1.154700538379251529018 29,-2/3}

The web is shown below with a period 6 orbit showing that the first ring of Dads decomposes into two groups of three each. This is a 'period-doubling' orbit so the center has period 3. This decomposition of the inner ring around Dad is canonical for 'twice-odd' regular polygons. However the second ring is period 12 and there is no doubling, as shown on third right below. So the kth ring of Dad has 6k hexagons and odd rings have decomposition and period doubling. The centers have periods 3k for odd rings and 6k for even and the step sequences are (2) (32), (332), etc.

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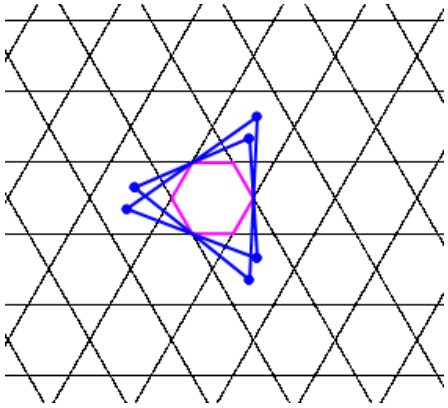
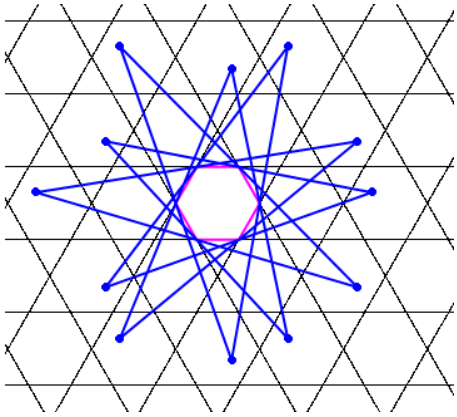
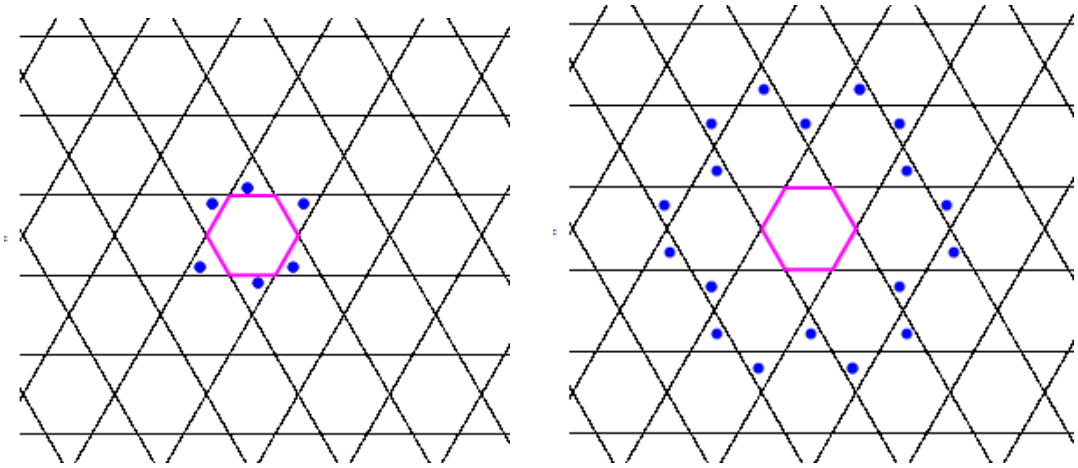
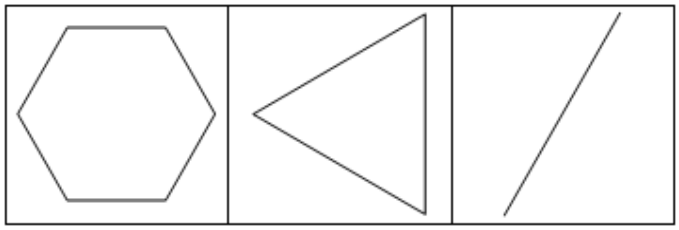




The triangular Moms have no doubling so their periods are the number of triangles in each ring, which is 6*(2k-1) for ring k. The first two rings are shown below.



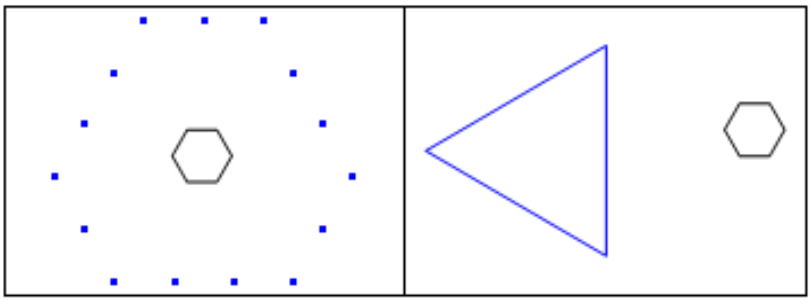
Projections
EulerPhi[6] = 2 so there are no non-redundant projections. The the remapping of the vertices are shown below.We will show examples of P1 and P2



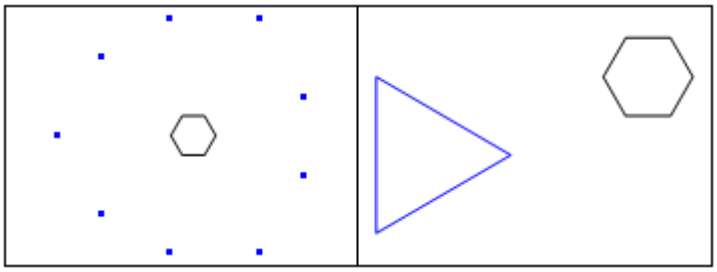
Example 1: **q1 = {-5.7,-0.8}** which is in the 3rd ring of triangles, so it has period 30. But the return map P1 has period 15 as shown below. As expected, the P2 projection builds a triangle.

Ind = IND[q1,100]; k = 20;

GraphicsGrid[{{
Graphics[{poly[Mom],Blue,Point[PIM[q1,k,1]]}],
Graphics[{poly[Mom],Blue,Line[PIM[q1,k,2]]}]}},
Frame->All]



Example 2: q1= {-7,0} is from the third ring of Dads so it is period 18 with period doubling and the P2 projection recognizes this.



Example 3: q1 = {-5,0} is from the second ring of Dads so it is period 12 with no doubling and the P2 projection is fixed as shown below