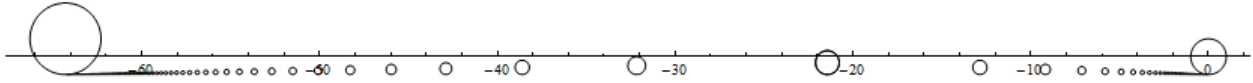


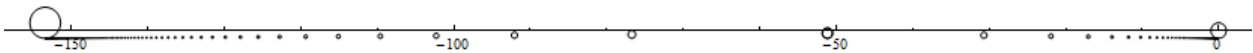
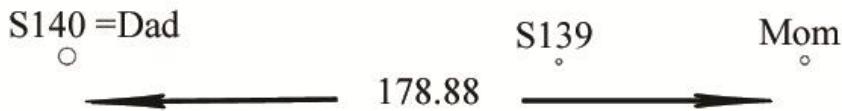
[GeneticsOfPolygons.org](http://GeneticsOfPolygons.org)

Summary of dynamics of the  
regular 281-gon:  $N = 281$

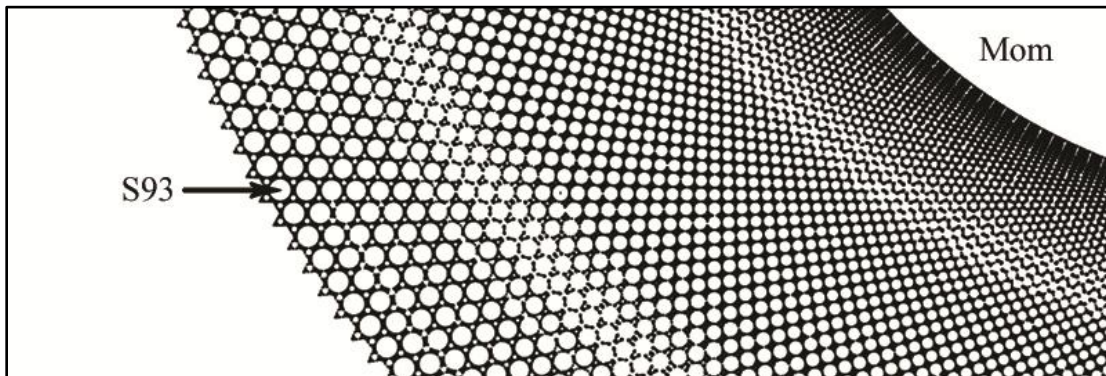
This is a token ‘large’  $4k+1$  prime, but as  $N$  grows in size the distinction between  $4k+1$  and  $4k+3$  primes is of lesser consequence. Below is the First Family for an intermediate  $4k+1$  prime,  $N = 101$ . Generating the matrices for these First Families becomes computationally intensive for large  $N$  because the size of the family is  $\text{Floor}[3N/2] - 3$  and most of these are  $2N$ -gons.



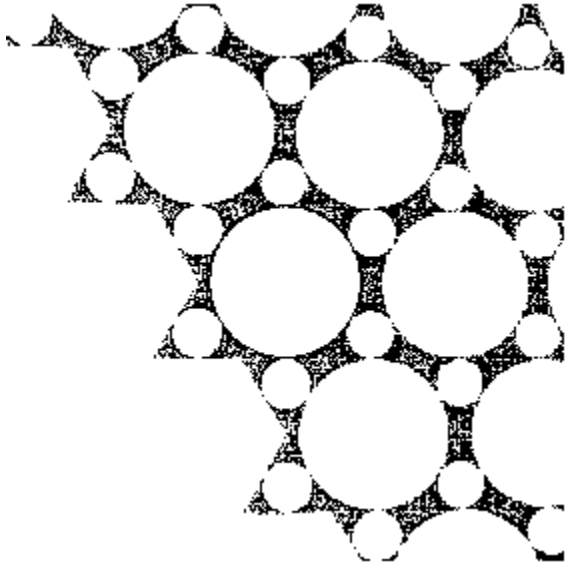
$N = 241$  shown below has 358 family members and takes about 5 minutes to generate and plot. As  $N$  increases, the star region grows while the scale shrinks.  $\text{GenScale}[281] \approx .000062$  so Mom[1] is microscopic and her local dynamics have little or no effect on the global dynamics-which are dominated by simple rotations about Mom – like a twist map on the unit circle with minimal perturbations. The amount of twist increases ‘smoothly’ with the distance from the origin so this system is close to being integrable. Dad is S[140] with maximal twist (for the inner star ) at  $\omega = 140/281$ . The period of the center of Dad is 281. There is a large gap between Dad and S[139] – which we call Big Brother. His period is also 281 since his orbit is entrained with the orbit of Dad. This is the only bud which is shared by Dad and Mom and he plays a pivotal role in the dynamics of the central region.



Below is the region around Mom which appears to be invariant. S[93] is the largest FirstFamily member of this region. The points shown here are all part of the orbit of  $p = \{-1.221862360, -.998696395\}$  which is in close proximity to S[10].

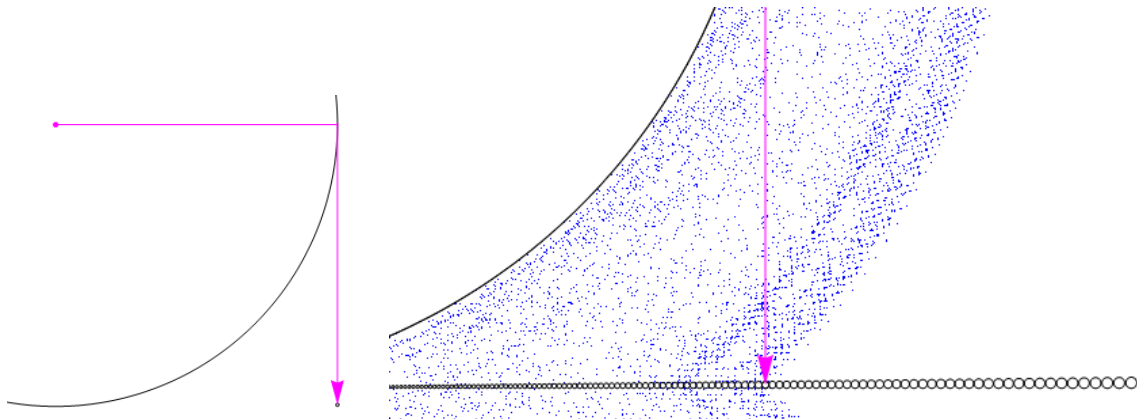


Below is an enlargement of the region around S[93]

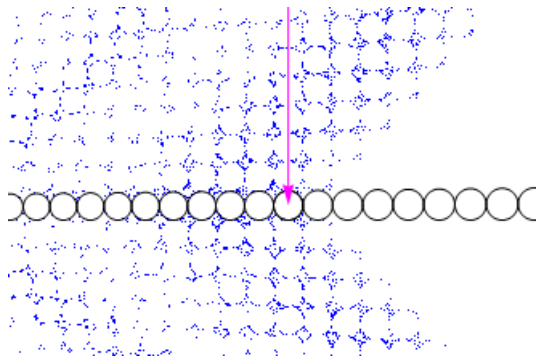


Below are close-ups of the region around Dad. For all prime regular polygons Dad is surrounded by an invariant ring. The nature of this ring varies between  $4k+1$  and  $4k+3$  primes. In all cases the bounding bud is bisected by the perpendicular from Dad as shown below. In  $4k+1$  primes this bounding bud is a  $2N$ -gon and for  $4k+3$  primes it is an  $N$ -gon. For  $N = 281$ , this  $2N$ -gon is DS[140]. Recall that the even steps for Dad are  $2N$ -gons and the odd steps are  $N$ -gons.

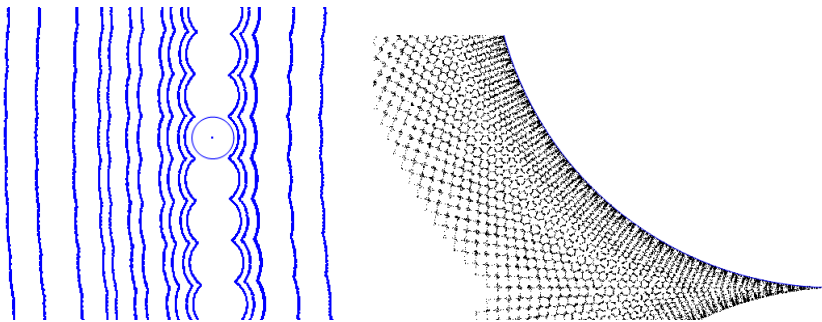
```
Show[Graphics[{AbsolutePointSize[5.0],poly[S[140]],poly[DS[140]],Magenta,
Arrow[{cDS[140]+{0,2},cDS[140]}],Magenta,
Line[{cS[140],cS[140]+{rDad,0}],Point[cS[140]]},PlotRange->{{left,right},{bottom,top}}]]
```



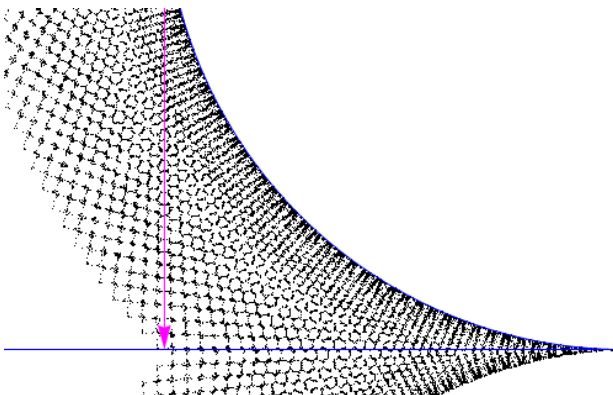
The points above and the detailed plot below are part of the orbit of  $p = \{-176.754572690, 2.34\}$ . The detailed plot below shows the region around DS[140]. It appears that the outer ring around Dad has the large-scale structure as the inner ring around Mom.



Big Brother is S [139] and also DS[278]. He is located at  $cS[139] \approx \{-59.631915403066097680920552, -0.3333472219236486001906099\}$ . It appears that he shares a similar local structure with Dad. On the left below is the large-scale structure around S[139] and on the right is the small-scale structure



The perpendicular once again bisects the last bud in the invariant ring surrounding S[139]. The intersection is at  $\{cS[139][[1]]-rS[139], cS[139][[2]]-hS[139]\} \approx \{-60.298516100149375514482447, -0.9999375039475275064779041\}$



It seems likely that the buds shown here would be part of the First Family for S[139]. In which case Dad and Mom are in for a surprise when Big Brother comes to visit for the first time.