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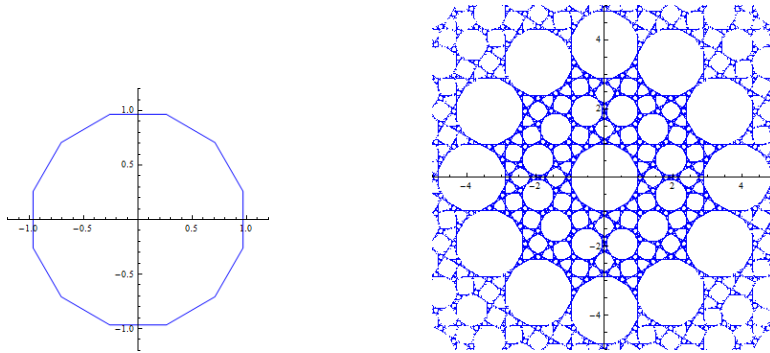
# Summary of dynamics of the regular dodecagon: $N = 12$

## Summary of results for regular dodecagon: N = 12

Multiple factors of N = 12 create a complex geometry, but the web structure is a simple fractal with dimension about 1.2513. This is similar to N = 5 and N = 8. Note that all of these have EulerPhi = 4 so the degree of the cyclotomic field extension is 4. This in turn implies that they have just 4 independent vertices (and only 2 independent projections).

To get an 'upright' Mom use same rotation scheme as N = 8:

```
Mac=Table[N[{Sin[2 Pi n/npoints],Cos[2 Pi n/npoints]},dplaces],{n,1, npoints}];
Mc=RotationTransform[-Pi/npoints][Mac];
Mom=RotateRight[Mc];Mc=Append[Mc,Mc[[1]]];
```

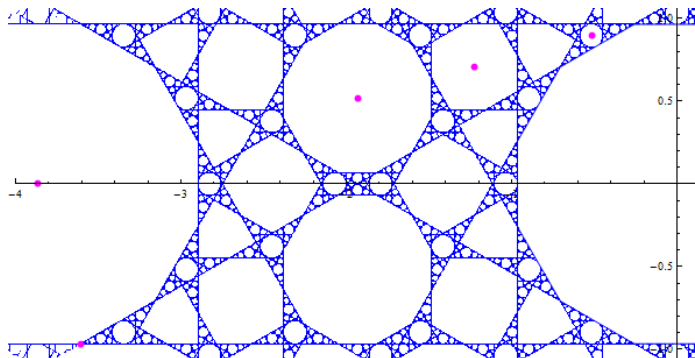


Using variants of FindC, we can locate centers of most major buds without using any further geometry. These 'hits' are given below. For non prime polygons, this is a 'black art'.

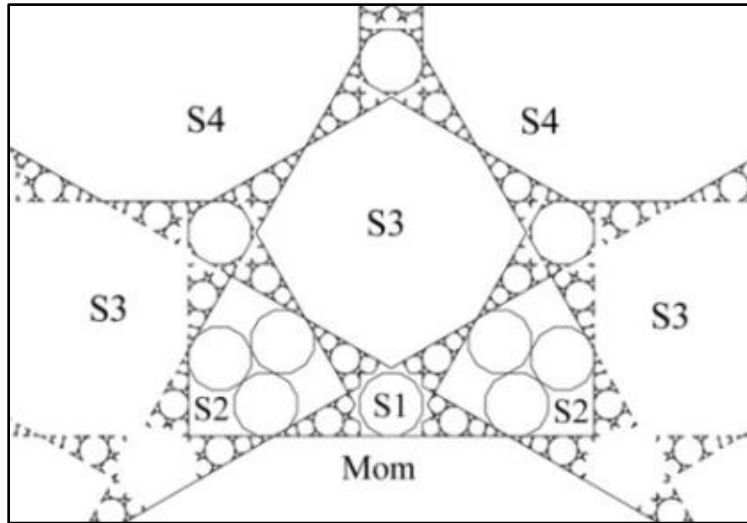
```
cS1 = {-0.5176380902050415246977976752, 0.8965754721680535241038910490}
cS3 = {-1.22474487139158904909864203735, 0.7071067811865475244008443621}
cS4 = {-1.9318516525781365734994863994, 0.5176380902050415246977976752}
cS5 (Dad) = {-3.8637033051562731469989727989029999999980768, 0}
Gen Star= {-3.6048842600537523846500739613, -0.96592582628906828674974319973}
```

```
GenScale = EvenScale[12]= 0.0717967697244908258902146340
```

The First Family is shown below in a web scan. The magenta dots are the 5 points listed above. S1 and S4 are normal 12-gons but S2 (period 6) is a non-regular hexagon like Helen from N = 9, and S3 (period 4) has 8 interwoven sides.

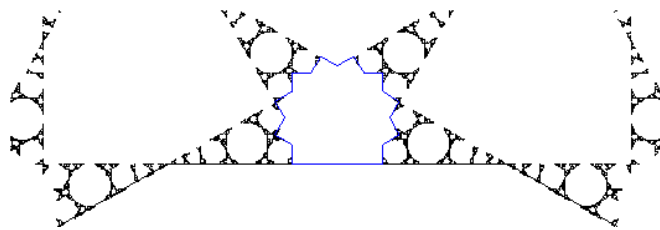


The detailed geometry is quite unique. Below is the top of Mom.

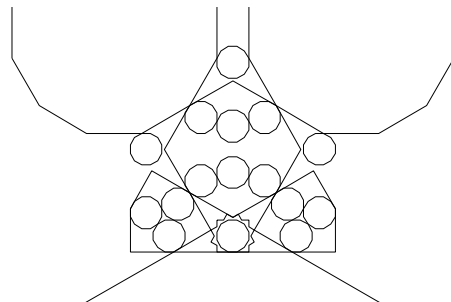


S1 is also Mom[1] and she presides over a self-similar generation 2 on the top of Mom[0], but the inner invariant region around Mom[1] is devoid of progeny, so S1[2] and S2[2] are missing. This means that subsequent generations on top of Mom[0] will have to form on the edges of the virtual Dad[1]'s. So even though Dad[1] and Mom[1] here are identical in size, they play different roles.

The 'halo' around S1 mimics the boundaries of large-scale invariant regions which occur for almost all N-gons, but this is the first known case where the interior of the invariant region is devoid of structure. S1 is period 12 and the halo is period 24. Neither has doubling.



We have drawn triplets of Dad[1]'s inside the S2's, to show the perfect fit. In the position shown above, S3 has center at  $\{0, \sqrt{2}\}$ . The interior of S3 extends the ring of virtual Dad[1]'s as shown below.



The chain of Moms at GenStar has ratio of periods which appears to approach 27. To find the centers of the Moms (=Dads) at GenStar, use CenterFromRadius

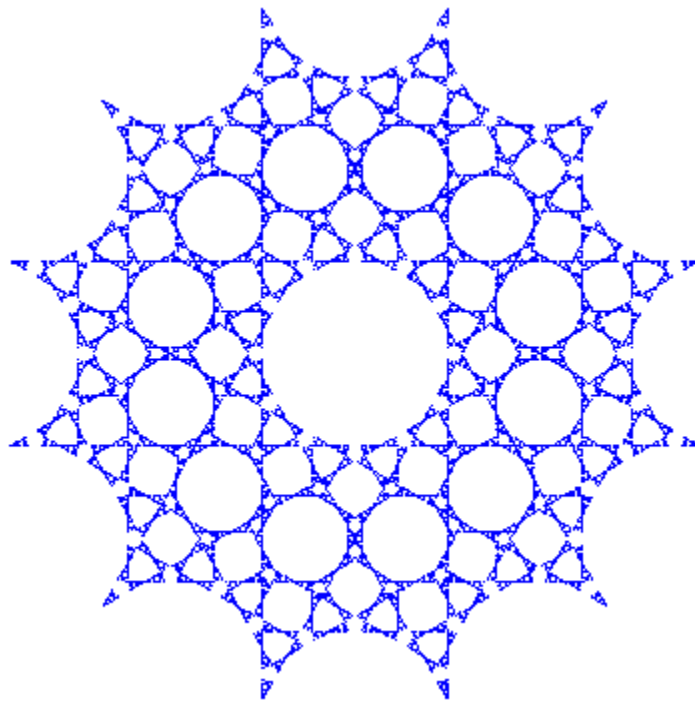
**CFR[r\_]:= (1-r)\*GenStar; cMom[k] := CFR[GenScale^k];**

Mom[k]	Period
Mom[1]	48
Mom[2]	504
Mom[3]	15,144
Mom[4]	388,104
Mom[5]	10,538,184
Mom[6]	284,779,464

Ratios of periods : 10.5 → 30.04 → 25.627 → 27.15 → 27.02

In the limit these Moms are dense in the star region. For example the first 50,000 points in the orbit of Mom[6] are shown below:

**Orbit = V[cMom[6], 50000]; Graphics[{AbsolutePointSize[1.0], Blue, Point[Orbit]}]**



The fractal dimension of the web is therefore  $\text{Log}[27]/\text{Log}[1/\text{GenScale}] \approx 1.2513$

## Projections

EulerPhi[12]/2 = 2 so there are only 2 non-redundant projections but we will look at all 6

```
IND[q_,maxn_]:=Module[{Ind=Range[maxn]},V[q,1];Ind[[1]]=Mod[cc+1,npoints,1];  
St=Vp[q,maxn];  
For[s=2,s<=maxn,s++,Ind[[s]]=Mod[Ind[[s-1]]+St[[s-1]],npoints,1]];  
Return[Ind];
```

```
PIM[q_,n_,j_]:=Module[{Pim=Range[n]},Pim[[1]]=q;  
For[s=2,s<=n,s++,Pim[[s]]=Pim[[s-1]]+2*(Sum[(-1)^k*Wc[[j]][[Ind[[k]]]],{k,2*s-3,2*s-2}]);  
Return[Pim];
```

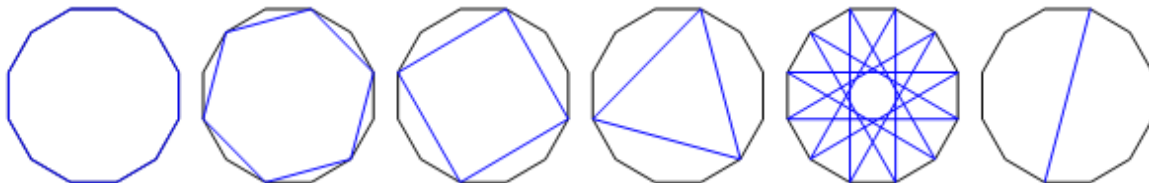
```
T1=Table[Mod[j,npoints,1],{j,1,npoints*1,1}];  
T2=Table[Mod[j,npoints,1],{j,1,npoints*2,2}];  
T3=Table[Mod[j,npoints,1],{j,1,npoints*3,3}];  
T4=Table[Mod[j,npoints,1],{j,1,npoints*4,4}];  
T5=Table[Mod[j,npoints,1],{j,1,npoints*5,5}];  
T6=Table[Mod[j,npoints,1],{j,1,npoints*6,6}];
```

```
w=Mom;Wc=Range[6];
```

```
Wc[[1]]=Table[w[[T1[[k]]]],{k,1,npoints}];  
Wc[[2]]=Table[w[[T2[[k]]]],{k,1,npoints}];  
Wc[[3]]=Table[w[[T3[[k]]]],{k,1,npoints}];  
Wc[[4]]=Table[w[[T4[[k]]]],{k,1,npoints}];  
Wc[[5]]=Table[w[[T5[[k]]]],{k,1,npoints}];  
Wc[[6]]=Table[w[[T6[[k]]]],{k,1,npoints}];
```

To see what we can expect, below are the 6 sets of remapped vertices:

```
GraphicsGrid[  
{Graphics[{poly[Mom], Blue, poly[Wc[[1]]]}], Graphics[{poly[Mom],Blue, poly[Wc[[2]]]}],  
Graphics[{poly[Mom],Blue, poly[Wc[[3]]]}], Graphics[{poly[Mom],Blue, poly[Wc[[4]]]}],  
Graphics[{poly[Mom],Blue, poly[Wc[[5]]]}], Graphics[{poly[Mom],Blue, poly[Wc[[6]]]}]}
```



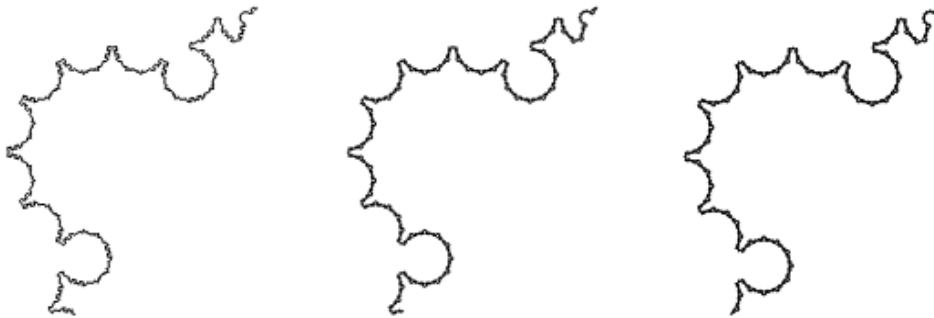
**Example:** We will find all possible projections for the center of Mom[6] which has period 284,779,464.

$q1 = cMom6 = \{-3.604883766288237300989394, -0.9659256939849972697455279636\}$

**Ind = IND[q1,1000000];**

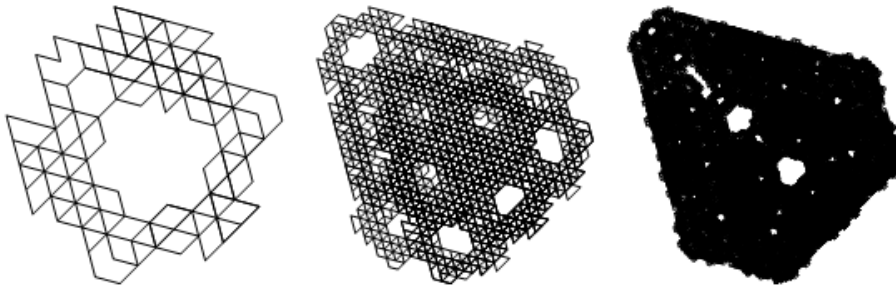
The P6 projection is constant so it is just a line. The only legitimate projection is P5. It is not surprising that the orbit of Mom[6] is (coarsely) self-similar. Below is a grid showing 600, 15,000 and 400,000 iterations. They are almost indistinguishable. The point q1 is at upper right.

**GraphicsGrid[{{Graphics[{Line[PIM[q1,600,5]}]}, Graphics[{Line[PIM[q1,15000,5]}]}, Graphics[{Line[PIM[q1,400000,5]}]}]}]**



Of course this self-similarity is reflected in all the projections. Below is are the triangles of P4

**GraphicsGrid[{{Graphics[{Line[PIM[q1,600,4]}]}, Graphics[{Line[PIM[q1,15000,4]}]}, Graphics[{Line[PIM[q1,400000,4]}]}]}]**

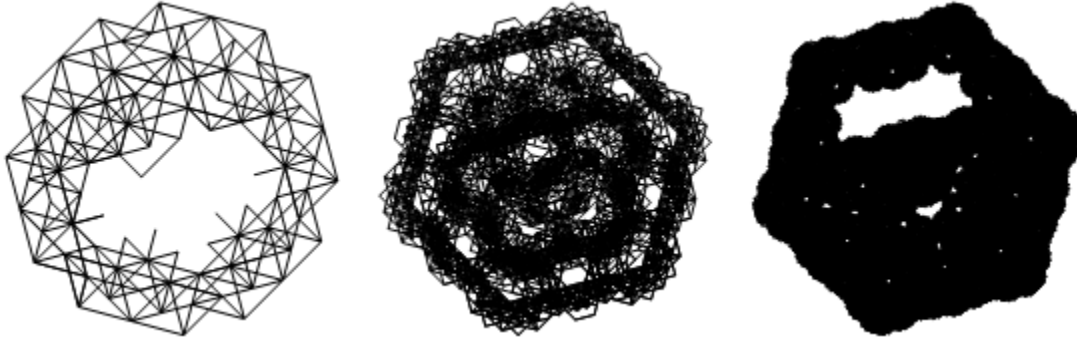


Now the squares of P3: **GraphicsGrid[{{Graphics[{Line[PIM[q1,600,3]}]}, Graphics[{Line[PIM[q1,15000,3]}]}, Graphics[{Line[PIM[q1,400000,3]}]}]}]**



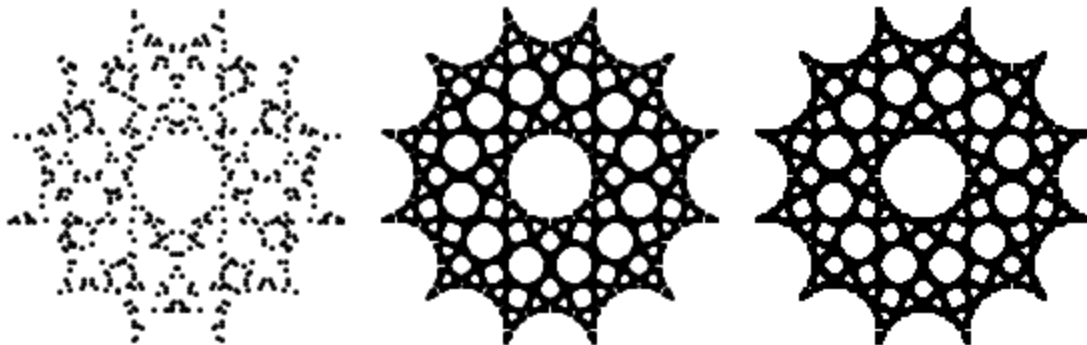
P2 is tightly woven hexagons

```
GraphicsGrid[{{Graphics[{Line[PIM[q1,600,2]}]},Graphics[{Line[PIM[q1,15000,2]}]},  
Graphics[{Line[PIM[q1,400000,2]}]}]}]
```



The null projection P1 is the return map  $\tau^2$  which is pretty dense so it is plotted below as points instead of lines. We can do this with other projections - especially when they are very large.

```
GraphicsGrid[{{Graphics[{Point[PIM[q1,600,1]}]},Graphics[{Point[PIM[q1,15000,1]}]},  
Graphics[{Point[PIM[q1,400000,1]}]}]}]
```



To get the full orbit of q1 instead of the return orbit, just take P1 again after dropping the first vertex in the vertex set Ind above. A plot of the 'even' and 'odd' orbits would look identical on this scale but they have no points in common, For small regions, we sometimes use both to fill in the missing points (but probably some symmetry algorithm would do the same thing.)