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Summery of dynamics of the half-circle

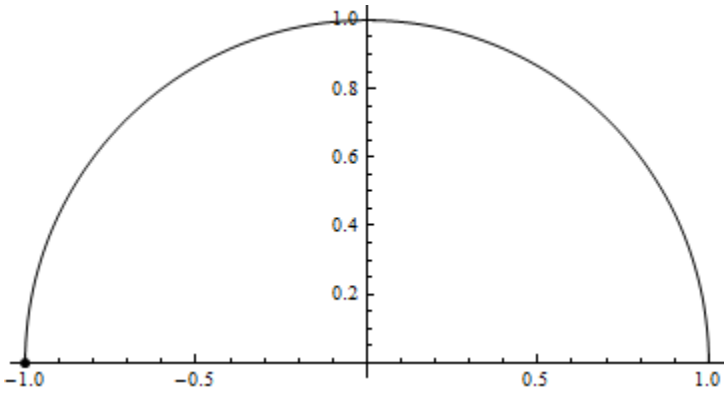
Summary of Dynamics of a 'HalfCircle': N = 100

D. Dolgopyat, B. Fayad, *Unbounded orbits for semicircular outer billiard.* Ann. Henri Poincare **10** (2009), 357–375. MR2511890 (2010d:37076)

The authors show that a semicircle has unbounded orbits that diverge in spiral patterns. This applies to related curves which are also 'sliced' circles.

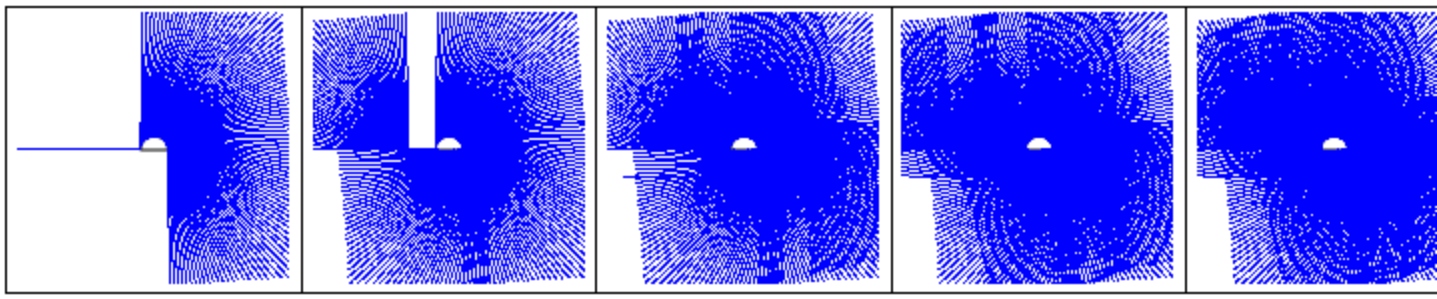
We will illustrate this continuous case with a polygonal 'half-circle'. Start with a regular polygon with 200 sides and cut it in half to get 100 sides as shown below.The notebook is NonRegular.nb

```
npoints = 200;dplaces = 30;Ma=Table[N[{Sin[2 Pi n/npoints],Cos[2 Pi
n/npoints]},dplaces],{n,1,npoints}];
Mb= Table[Ma[[k]],{k,100,200}];Mc = RotationTransform[3*Pi/2][Mb];
Mom = Mc; npoints= Length[Mc];  dplaces =30;


Graphics[{poly[Mom],AbsolutePointSize[5.0], Point[Mom[[1]]]}, Axes->True]
```



Below is a web scan showing the first 5 iterations. These are very dense webs indicating unstable dynamics. Web lines cross each other in a nearly random fashion. The 'spiraling' behavior can already be observed by the level 3 web.

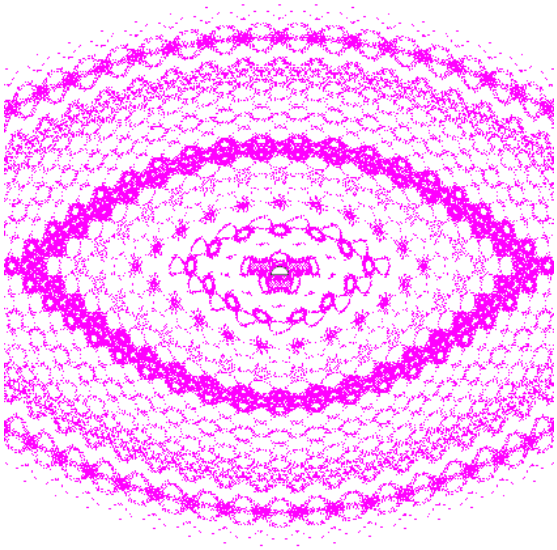
```
Gr[depth_]:=Show[Graphics[{AbsolutePointSize[1.0],poly[Mom],Blue,Point[WebPoints[.0
7,10,depth]]},

PlotRange->{{left,right},{bottom,top}}]];

GraphicsGrid[{{Gr[0],Gr[1],Gr[2],Gr[3], Gr[4]}},Frame->All]
```



Pick an arbitrary probe point close to the x axis: **q1= {-1.6,.002357}; K = V[q1,100000]; box[{0,0},30];**

The first 100,000 points in the orbit of q1 are shown below. It is no surprise that points spiral outwards.

Show[Graphics[{AbsolutePointSize[1.0],poly[Mom],Blue, AbsolutePointSize[1.0],Magenta, Point[K]},
PlotRange->{{left,right},{bottom,top}}]]

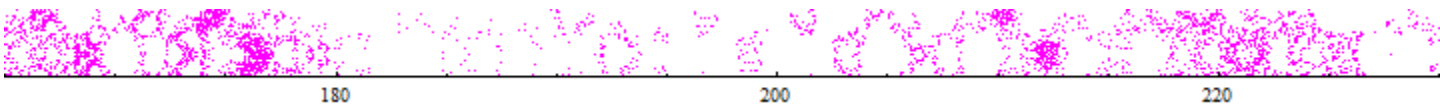


Using q1 from above as a probe, explore one 'strip' to see how the web developes

t**op =3; bottom = 0, left = -1; right = 2000; Datumcrop[q1,20,1000000, "temp"]**
After 20 million iterations, this orbit returns 281477 points in the file Wcrop. The extent of this orbit is about 400 as shown below

Below is a close up of this strip to distance of 30

Below is the 'void' in the vicinity of 200. This might be the beginning of a new cycle. Most unbounded orbits generate semi-periodic web structures.It is too early to conjecture that q1 has an unbounded orbit, but clearly there is no shortage of points with divergent orbits.



Projections

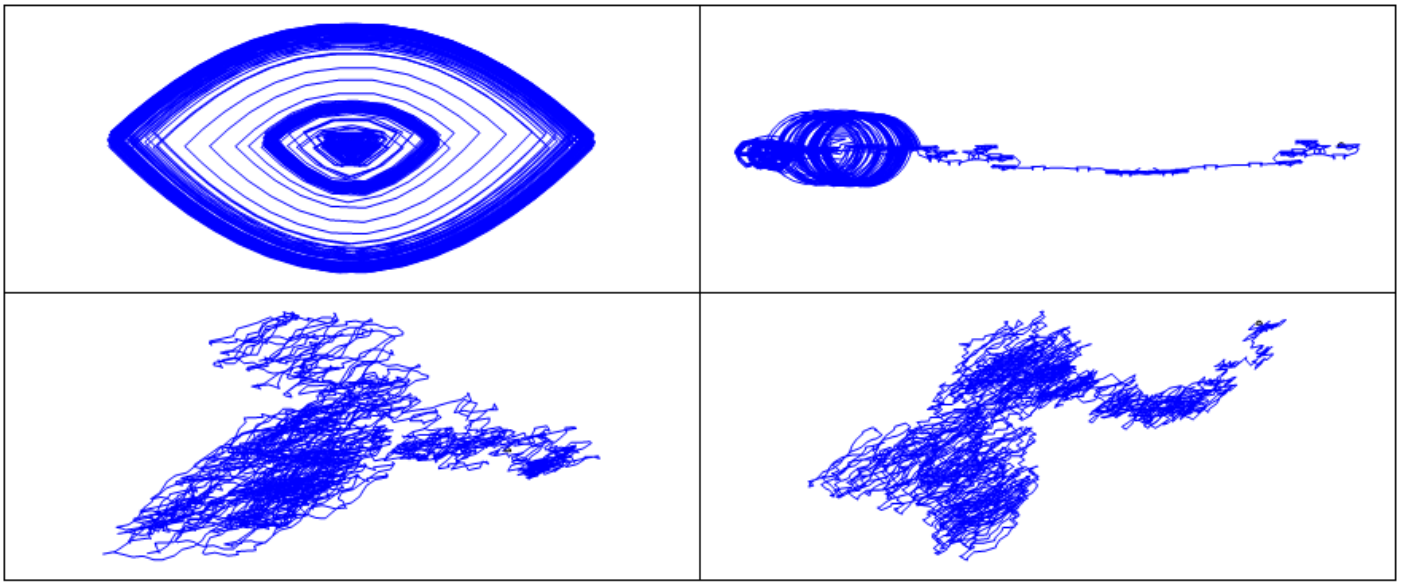
There are 50 non-redundant projections but most of them yield similar orbits. Below are some samples

GraphicsGrid[{Table[Graphics[poly[Wc[[k]]]],{k,1,50,5}]},Frame->All]

Ind = IND[q1,50000]; k = 5000; (* use same point as above*)

Px=Table[Graphics[{poly[Mom],Blue,Line[PIM[q1,k,j]]}],{j,1,50}];

GraphicsGrid[{{Px[[1]],Px[[2]]},{Px[[25]],Px[[47]]}},Frame->All]



As orbits spiral outwards, the dynamics are dominated by rotations. In P2 above, we can see the onset of this rotational behavior. These rotations can be filtered out and the result is 'Pinwheel' projections which can provide valuable information about the evolution of an orbit.